Remarks on soft $\omega$-closed sets in soft topological spaces

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Abstract: The paper introduces soft $\omega$-closed sets in soft topological spaces and establishes the relation between other existing generalised closed sets in soft topological spaces. It derives the basic properties of soft $\omega$-closed sets. As an application it proves that a soft $\omega$-closed set in a soft compact space is soft compact.

Key Words: soft open set, soft closed set, soft $\omega$-closed set and soft $\omega$-open set.

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1. Introduction

The soft set introduced by Molodtsov [8] is applied in many fields such as economics, engineering, social science, medical science etc. It is used as a tool for dealing with uncertain objects. It laid the platform for further research involving soft sets. The topological structures of set theories dealing with uncertainties was introduced by Chang [2]. Shabir and Naz [9] defined soft topological spaces over an universe. They investigated the basic properties of soft topological spaces. Aygunoglu et.al. [1] also discussed the properties of soft topological spaces. Chen [3] introduced soft semi-open and soft semi-closed sets. Topology is considered to be one of the main branches of Mathematics along with algebra and analysis. Levine [6] has introduced generalised closed sets in topology in order to extend the properties of closed sets to a larger family. In the recent past there has been considerable research in the study of various forms of generalised closed sets. Kannan [5] has introduced soft generalised closed set in soft topological spaces. In this paper soft $\omega$-closed sets are introduced in soft topological spaces and some of its basic properties are discussed. Soft $\omega$-open sets are also defined and the necessary and sufficient
condition for a soft set to be soft $\omega$-closed and soft $\omega$-open are derived. The soft $\omega$-closed set concept has been extended to subspaces.

2. Preliminaries

Definition 2.1. [8] Let $X$ be an initial universe and $E$ be a set of parameters. Let $P(X)$ denote the power set of $X$ and $A$ be a non-empty subset of $E$. A pair $(F,A)$ is called a soft set over $X$, where $F$ is a mapping given by $F : A \rightarrow P(X)$. The family of all soft sets $(F,A)$ over $X$ is denoted by $SS(X,A)$ for two soft sets $(F,A)$ and $(G,A)$ over a common universe $X$, $(F,A)$ is said to be soft subset of $(G,A)$ if (i) $F(p) \subseteq G(p)$ for all $p \in A$. Symbolically it is written as $(F,A) \subseteq (G,A)$.

The pair $(F,A)$ and $(G,A)$ over a common universe $X$, $(F,A)$ is said to be soft subs set of $(G,A)$ if (i) $F(p) \subseteq G(p)$ for all $p \in A$. Symbolically it is written as $(F,A) \subset (G,A)$. The pair $(F,A)$ and $(G,A)$ are soft equal if $(F,A) \subseteq (G,A)$ and $(G,A) \subseteq (F,A)$. Symbolically it is written as $(F,A)=(G,A)$ [8]

Definition 2.2. [8] Let $I$ be an arbitrary index set and $\{(F_i,A) : i \in I\} \subseteq SS(X,A)$. The soft union of these soft sets is the soft set $(F,A) \in SS(X,A)$ where the map $F : A \rightarrow P(X)$ is defined as $F(p) = \bigcup\{F_i(p) : i \in I\}$ for every $p \in A$ and denoted as $(F,A) = \bigcup\{(F_i,A) : i \in I\}$.

Definition 2.3. [8] Let $I$ be an arbitrary index set and $\{(F_i,A) : i \in I\} \subseteq SS(X,A)$. The soft intersection of these soft sets is the soft set $(F,A) \in SS(X,A)$ where the map $F : A \rightarrow P(X)$ is defined as $F(p) = \bigcap\{F_i(p) : i \in I\}$ for every $p \in A$ and denoted as $(F,A) = \bigcap\{(F_i,A) : i \in I\}$.

Definition 2.4. [8] A soft set $(F,A)$ over $X$ is said to be null soft set denoted by $\phi$ if for all $p \in A$ $F(p) = \phi$. A soft set $(F,A)$ over $X$ is said to be an absolute soft set denoted by $\bar{A}$ if for all $e \in A$, $F(e) = X$.

Definition 2.5. [8] Let $Y$ be a non-empty subset of $X$ then $\tilde{Y}$ denotes the soft set $(Y,E)$ over $X$ for which $Y(e)=Y$, for all $e \in E$. In particular $(X,E)$ will be denoted by $X$.

Definition 2.6. [8] The difference $(H,E)$ of two soft sets $(F,E)$ and $(G,E)$ over $X$ denoted by $(F,E) \setminus (G,E)$ is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

Definition 2.7. [9] The relative complement of a soft subset $(F,E)$ is denoted by $(F,E)^c$ and is defined by $(F,E)^c = (F^c,E)$ where $F^c : E \rightarrow P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e \in E$.

Definition 2.8. [9] Let $\tau$ be the collection of soft sets over $X$, then $\tau$ is said to be a soft topology on $X$ if

(i) $\phi, X \in \tau$

(ii) If $(F,E), (G,E) \in \tau$ then $(F,E) \cap (G,E) \in \tau$

(iii) If $\{(F_i,E)\}_{i \in I} \in \tau$, then $\bigcup_{i \in I}(F_i,E) \in \tau$
The triplet $(X, \tilde{\tau}, E)$ is called a soft topological space over $X$. Every member of $\tilde{\tau}$ is called a soft open set. A soft set $(F, E)$ is called soft closed in $X$ if $(F, E)^c \in \tilde{\tau}$. The soft closure of a soft set over $X$ is defined as the intersection of all soft closed supersets of $(F, E)$ and is denoted as $(F, E)\bar{c}$ and it is the smallest soft closed set over $X$ containing $(F, E)$. The soft interior of the soft set $(F, E)$ is defined as union of all soft open subsets of $(F, E)$ and is denoted as $(F, E)^0$ and it is the largest soft open set over $X$ which is contained in $(F, E)$.

**Theorem 2.9.** [9] Let $(X, \tilde{\tau}, E)$ be a soft topological space over $X$, $(F, E)$ and $(G, E)$ be soft sets over $X$. Then

(i) $\tilde{\phi} = \tilde{\phi}$ and $\tilde{X} = \tilde{X}$

(ii) $(F, E) \subseteq (F, E)$

(iii) $(F, E)$ is a soft closed set if and only if $(F, E) = (F, E)\bar{c}$

(iv) $(F, E) = (F, E)\bar{c}$

(v) $(F, E) \subseteq (G, E)$ implies $(F, E) \subseteq (G, E)$

(vi) $(F, E) \sqcup (G, E) = (F, E) \sqcup (G, E)$

(vii) $(F, E) \sqcap (G, E) \subseteq (F, E) \sqcap (G, E)$

**Theorem 2.10.** [3] Let $(X, \tilde{\tau}, E)$ be a soft topological space over $X$, $(F, E)$ and $(G, E)$ be soft sets over $X$. Then

(i) $\tilde{\phi}^0 = \tilde{\phi}$ and $\tilde{X}^0 = \tilde{X}$

(ii) $(F, E)^0 \subseteq (F, E)$

(iii) $(F, E)$ is a soft open set if and only if $(F, E) = (F, E)^0$

(iv) $((F, E)^0)^0 = (F, E)^0$

(v) $(F, E) \subseteq (G, E)$ implies $(F, E)^0 \subseteq (G, E)^0$

(vi) $((F, E) \sqcap (G, E))^0 = (F, E)^0 \sqcap (G, E)^0$

(vii) $((F, E) \sqcup (G, E))^0 \supseteq F, E)^0 \sqcup (G, E)^0$

**Theorem 2.11.** [3] Let $(X, \tilde{\tau}, E)$ be a soft topological space over $X$, $(F, E)$ and $(G, E)$ are soft sets over $X$. Then

(i) $((F, E)^c)^0 = ((F, E))^c$

(ii) $(F, E)^c = ((F, E)^0)^c$

**Definition 2.12.** [8] Let $(F, E)$ be a soft set over $X$ and $Y$ be non-empty subset of $X$. Then the soft subset of $(F, E)$ over $Y$ denoted by $(Y, E)$ is defined as $Y_{F(e)} = Y \cap F(e)$ for all $e \in E$. In other words $(Y, E) = \tilde{Y} \cap F(e)$.
Definition 2.13. Let \((X, \tilde{\tau}, E)\) be a soft topological space over \(X\) and \(Y\) be a nonempty subset of \(X\). Then \(\tilde{\tau}_Y = \{(Y_F, E) : (F, E) \in \tilde{\tau}\}\) is said to be the soft relative topology on \(Y\) and \((Y, \tilde{\tau}_Y)\) is called a soft subspace of \((X, \tilde{\tau})\). In fact \(\tilde{\tau}_Y\) is a soft topology on \(Y\).

Theorem 2.14. Let \((Y, \tilde{\tau}_Y)\) be a soft subspace of a soft topological space \((X, \tilde{\tau})\) and \((F, E)\) be a soft set over \(X\) then

(i) \((F, E)\) is soft open in \(Y\) if and only if \((F, E) = \tilde{\tau}_Y \sqcap (G, E)\) for some \((G, E) \in \tilde{\tau}\).

(ii) \((F, E)\) is soft closed in \(Y\) if and only if \((F, E) = \tilde{\tau}_Y \sqcap (G, E)\) for some soft closed set \((G, E)\) in \(X\).

Proposition 2.15. Let \((A, E)\) and \((G, E)\) are soft sets over \(X\) then

(i) \(((A, E) \sqcup (G, E))^c = (F, E)^c \sqcap (G, E)^c\)

(ii) \(((A, E) \sqcap (G, E))^c = (F, E)^c \sqcup (G, E)^c\)

Definition 2.16. Let \((X, \tilde{\tau}, E)\) be a soft topological space.

(i) A family \(C = \{(F_i, E) : i \in I\}\) of soft open sets in \(X\) is called a soft open cover of \(X\), if \(\sqcup_{i \in I} (F_i, E) = \tilde{\tau}_X\). A finite subfamily of soft open cover of \(C\) of \(X\) is called a finite sub cover of \(X\), it it is also a soft open cover of \(X\).

(ii) \(X\) is called soft compact if every soft open cover of \(X\) has a finite subcover.

Definition 2.17. Let \((X, \tilde{\tau}, E)\) be a soft topological space, \((F, E)\) and \((G, E)\) are soft closed sets in \(X\) such that \((F, E) \sqcap (G, E) = \tilde{\phi}\). If there exists soft open sets \((A, E)\) and \((B, E)\) such that \((A, E) \sqsubseteq (A, E), (G, E) \sqsubseteq (B, E)\) and \((A, E) \sqcap (B, E) = \tilde{\phi}\), then \(X\) is called a soft normal space.

Theorem 2.18. Let \((X, \tilde{\tau}, E)\) be a soft topological space if \((F, E)\) is a soft closed set in \(X\), then \((F, E)\) is soft compact.

Definition 2.19. A soft set in a soft topological space is said to be soft semi closed if its relative complement is soft semi open.

Theorem 2.20. A soft subset \((B, E)\) in a soft topological space is soft semi closed if and only if \((B, E) \sqsubseteq (A, E)^0\).

Theorem 2.21. Let \(\{(A_\alpha, E)\}_{\alpha \in \Delta}\) be a collection of soft semi open sets in a soft topological space. Then \(\bigcup_{\alpha \in \Delta} (A_\alpha, E)\) is soft semi open.

Definition 2.22. A soft set in a soft topological space is said to be soft semi closed if its relative complement is soft semi open.

Theorem 2.23. A soft subset \((B, E)\) in a soft topological space is soft semi closed if and only if \(\overline{((B, E)^0)} \sqsubseteq (B, E)\).

Remark 2.24. Every soft closed set in a soft topological space is soft semi closed. The converse is not true.

Theorem 2.25. Let \(\{(B_\alpha, E)\}_{\alpha \in \Delta}\) be a collection of soft semi closed sets in a soft topological space. Then \(\bigcap_{\alpha \in \Delta} (B_\alpha, E)\) is soft semi closed.
3. soft $\omega$-closed sets

In this section soft $\omega$-closed set is introduced and some of its basic properties are derived. The necessary and sufficient condition for a soft set to be soft $\omega$-closed is stated and proved.

**Definition 3.1.** A soft set $(A,E)$ is called soft $\omega$-closed in a soft topological space $(X,\tilde{\tau},E)$, if $(A,E) \subseteq (G,E)$ whenever $(A,E) \subseteq (G,E)$ and $(G,E)$ is soft semi-open in $X$.

**Proposition 3.2.** Every soft closed is soft $\omega$-closed.

**Proof:** Let $(F,E)$ be a soft closed set and $(G,E)$ be a soft semi-open set in $X$ containing $(F,E)$. Then $(F,E) = (F,E) \subseteq (G,E)$. Hence $(F,E)$ is soft $\omega$-closed. $\square$

**Remark 3.3.** The converse of the proposition 3.2 is not true.

**Example 3.4.** Let $X = \{x, y, z\}, E = \{a, b\}$ The soft set $(F,E)$ is defined as $F(a) = \{x\}, F(b) = \{y\}$ and the soft set $(G,E)$ is defined as $G(a) = \{x, y\}$, $G(b) = \{y, z\}$ and $\tilde{\tau} = \{\phi, X, (F,E), (G,E)\}$. The soft set $(H,E)$ defined by $H(a) = \{z\}, H(b) = \{\phi\}$ is soft $\omega$-closed but not soft closed.

**Proposition 3.5.** Every soft $\omega$-closed set is soft $g$-closed.

**Proof:** Let $(H,E)$ be a soft $\omega$-closed set and $(U,E)$ be a soft open set containing $(H,E)$. Since every soft open set is soft semi-open, $(H,E) \subseteq (U,E)$. Hence $(H,E)$ is soft $\omega$-closed. $\square$

**Remark 3.6.** The converse of the proposition 3.5 is not true.

**Example 3.7.** Let $X = \{x, y, z\}, E = \{a, b\}$ The soft set $(F,E)$ is defined as $F(a) = \{x\}, F(b) = \{y\}$ and the soft set $(G,E)$ is defined as $G(a) = \{x, y\}$, $G(b) = \{y, z\}$ and $\tilde{\tau} = \{\phi, X, (F,E), (G,E)\}$. The soft set $(H,E)$ defined by $H(a) = \{x, z\}, H(b) = \{x, y\}$ is soft $g$-closed but not soft $\omega$-closed.

**Theorem 3.8.** If $(A,E)$ and $(B,E)$ are soft $\omega$-closed sets then $(A,E) \cup (B,E)$ is also soft $\omega$-closed.

**Proof:** Let $(U,E)$ be a soft semi-open set containing $(A,E) \cup (B,E)$. Then $(A,E) \subseteq (U,E)$ and $(B,E) \subseteq (U,E)$. Since $(A,E)$ and $(B,E)$ are soft $\omega$-closed sets $(A,E) \subseteq (U,E), (B,E) \subseteq (U,E)$. Hence $(A,E) \cup (B,E) = (A,E) \cup (B,E) \subseteq (U,E)$. $\square$

**Proposition 3.9.** If $(A,E)$ is soft $\omega$-closed and $(A,E) \subseteq (B,E) \subseteq \overline{(A,E)}$ then $(B,E)$ is also soft $\omega$-closed.

**Proof:** Suppose $(A,E)$ is soft $\omega$-closed and $(A,E) \subseteq (B,E) \subseteq \overline{(A,E)}$. Let $(B,E) \subseteq (U,E)$ and $(U,E)$ be soft semi-open, then $(A,E) \subseteq (U,E)$. Since $(A,E)$ is soft $\omega$-closed, $(A,E) \subseteq (U,E)$ and $(B,E) \subseteq (A,E) \subseteq (U,E)$. Hence $(B,E)$ is soft $\omega$-closed. $\square$
Theorem 3.10. If a set $(A,E)$ is soft $\omega$-closed in $X$ then $(\overline{A}, E) - (A, E)$ contains only null soft closed set.

Proof: Suppose $(A,E)$ is soft $\omega$-closed in $X$ and $(F,E)$ be a soft closed set such that $(F, E) \subseteq (\overline{A}, E) - (A, E)$ Since $(F,E)$ is soft closed its relative complement is soft open , $(F,E) \subseteq (A, E)^c$. Thus $(A, E) \subseteq (F, E)^c$. Consequently $(A, E) \subseteq (F, E)^c$. Therefore $(F, E) \subseteq ((A,E))^c$. Hence $(F, E) = \phi$ and thus $(\overline{A}, E) - (A, E)$ contains only null soft closed set. \qed

Theorem 3.11. A soft set $(A,E)$ is soft $\omega$-closed if and only if $(\overline{A}, E) - (A, E)$ contains only null soft semi-closed set.

Proof: Suppose that $(A,E)$ is soft $\omega$-closed in $X$ let $(F,E)$ be a soft semi-closed set such that $(F, E) \subseteq (A, E) - (A, E)$. Since $(F,E)$ is soft semi-closed its relative complement is soft semi-open with $(F,E) \subseteq (A, E)^c$. Thus $(A, E) \subseteq (F, E)^c$. Consequently $(A, E) \subseteq (F, E)^c$. Therefore $(F, E) \subseteq ((A,E))^c$. Hence $(F, E) = \phi$ and thus $(\overline{A}, E) - (A, E)$ contains only null soft semi-closed set. Conversely suppose that $(\overline{A}, E) - (A, E)$ contains only null soft semi-closed set. Let $(A, E) \subseteq (G, E)$ and $(G, E)$ be soft semi-open. If $(A,E)$ is not a subset of $(G,E)$ then $(A, E) \cap (G, E)^c$ is a non null soft semi-closed subset of $(\overline{A}, E) - (A, E)$ since any soft closed set is soft semi-closed and arbitrary intersection of soft semi-closed sets is soft semi-closed set [3] which is a contradiction. Thus $(A, E) \subseteq (G, E)$ and hence $(A,E)$ is soft $\omega$-closed. \qed

Theorem 3.12. If $(A,E)$ is soft semi-open and soft $\omega$-closed then $(A,E)$ is soft closed.

Proof: Since $(A,E)$ is soft semi-open and soft $\omega$-closed, $(\overline{A}, E) \subseteq (A, E)$. Hence $(A,E)$ is soft closed. \qed

Definition 3.13. The intersection of all soft semi open sets containing $(A,E)$ is called the semi-kernel of $(A,E)$ and is denoted as $\text{sker}(A,E)$.

Theorem 3.14. A soft set $(A,E)$ of a soft topological space $X$ is soft $\omega$-closed if and only if $(\overline{A}, E) \subseteq \text{sker}(A,E)$.

Proof: The first part follows from the definition of $\text{sker}(A,E)$. Conversely let $(\overline{A}, E) \subseteq \text{sker}(A,E)$. If $(U,E)$ is any soft semi-open set containing $(A,E)$, then $(A,E) \subseteq \text{sker}(A,E) \subseteq (U, E)$. Therefore $(A,E)$ is soft $\omega$-closed. \qed

Theorem 3.15. Let $(A,E)$ be a soft $\omega$-closed set in $X$. Then $(A,E)$ is soft closed if and only if $(\overline{A}, E) - (A, E)$ is soft semi-closed.

Proof: Suppose $(A,E)$ is soft $\omega$-closed which is also soft closed. Then $(\overline{A}, E) = (A, E)$ and so $(\overline{A}, E) - (A, E) = \phi$ which is soft semi-closed.
Conversely since \((A, E)\) is soft \(\omega\)-closed the by theorem 3.11 \(\overline{(A, E)} = (A, E)\) contains no non null soft semi-closed. But \(\overline{(A, E)} = (A, E)\) is soft semi-closed set. This implies that \(\overline{(A, E)} - (A, E) = \emptyset\). That is \((A, E)\) is soft closed. \(\square\)

**Theorem 3.16.** Let \((X, \tilde{\tau}, E)\) be a soft topological space and \(Y \subseteq Z \subseteq X\) be non-empty subsets of \(X\). If \(\tilde{Y}\) is a soft \(\omega\)-closed set relative to \((Z, \tilde{\tau}_Z)\) and \(\tilde{Z}\) is a soft \(\omega\)-closed set relative to \((X, \tilde{\tau})\), then \(\tilde{Y}\) is soft \(\omega\)-closed relative to \((X, \tilde{\tau})\).

**Proof:** Let \(\tilde{Y} \subseteq (F, E)\), \((F, E)\) is soft semi-open in \(X\). Since \(Y\) is a subset of \(Z\), \(\tilde{Y} \subseteq \tilde{Z}\) then \(Y \subseteq \tilde{Z} \cap (F, E)\). Since \(\tilde{Y}\) is soft \(\omega\)-closed relative to \((Z, \tilde{\tau}_Z)\) and \(\tilde{Z} \cap (F, E)\) is a soft semi-open set in \((Z, \tilde{\tau}_Z)\), \(\tilde{Z} \cap (F, E)\) represents the soft closure of \(\tilde{Y}\) with respect to the relative topology \((Z, \tilde{\tau}_Z)\). It follows that \(\tilde{Y} \subseteq \tilde{Z} \cap (F, E)\) and \(\tilde{Y} \subseteq (F, E)\). Hence \(\tilde{Z} \cap (F, E)\) is soft semi-open in \((Z, \tilde{\tau}_Z)\). Since \(Z\) is subset of \(X\), \(\tilde{Z} \subseteq \tilde{X}\). So \(\tilde{Z} \subseteq (F, E)\) and \((F, E)\) is soft semi-open in \(X\). Since \(\tilde{Z}\) is soft \(\omega\)-closed set relative to \(X\) and \(\tilde{Y} \subseteq \tilde{Z}, \tilde{Y} \subseteq (F, E)\). Therefore \(\tilde{Y} \subseteq (F, E)\) since \(\tilde{Y} \subseteq (F, E)\).

**Corollary 3.17.** If \((A, E)\) is a soft \(\omega\)-closed set and \((F, E)\) is a soft closed set in \(X\) then \((A, E) \cap (F, E)\) is soft \(\omega\)-closed set in \(X\).

**Proof:** \((A, E) \cap (F, E)\) is a soft closed set in \((A, E)\). By the Theorem 3.16 \((A, E) \cap (F, E)\) is soft \(\omega\)-closed in \(X\).

**Theorem 3.18.** Let \((X, \tilde{\tau}, E)\) be a soft topological space and \(Y \subseteq X\), \((F, E)\) be a soft set in \(Y\) such that it is \(\omega\)-closed in \(X\). Then \((F, E)\) is soft \(\omega\)-closed relative to \((Y, \tilde{\tau}_Y)\)

**Proof:** Let \((F, E) \subseteq \tilde{Y} \cap (G, E)\) and \((G, E)\) is soft semi-open in \(X\). Then \((F, E) \subseteq (G, E)\) and hence \((F, E) \subseteq (G, E)\) Hence \(\tilde{Y} \cap (F, E) \subseteq \tilde{Y} \cap (G, E)\).

**Theorem 3.19.** In a soft topological space \(SSO(X) = SC(X)\) if and only if every soft set over \(X\) is a soft \(\omega\)-closed set in \(X\). \(SSO(X)\) represents the collection of all soft semi-open sets in \(X\) and \(SC(X)\) represents the collection of all soft closed sets in \(X\).

**Proof:** Suppose that \(SSO(X) = SC(X)\). Let \((A, E)\) be a soft set of \(X\) such that \((A, E) \subseteq (G, E)\) where \((G, E) \in SSO(X)\). Then \((G, E) = (G, E)\). Also \((A, E) \subseteq (G, E)\). Hence \((A, E)\) is soft \(\omega\)-closed. Conversely suppose that every subset of \(X\) is soft \(\omega\)-closed. Let \((G, E) \in SSO(X)\) Since \((G, E) \subseteq (G, E)\), \((G, E) \subseteq (G, E)\) Thus \((G, E) = (G, E)\) and \((G, E) \in SC(X)\). Therefore \(SSO(X) \subseteq SC(X)\). If \((G, E) \in SC(X)\) then \((G, E)^c\) is soft open and hence soft semi-open. Therefore \((G, E)^c \subseteq (SC(X))^c \subseteq SC(X)\) and hence \((G, E) \in SSO(X)\). Thus \(SSO(X) = SC(X)\)
4. soft $\omega$-open sets

**Definition 4.1.** A soft set $(A,E)$ is called a soft $\omega$-open in a soft topological space $(X,\tilde{\tau},E)$ if the relative complement of $(A,E)$ is soft $\omega$-closed in $X$.

**Theorem 4.2.** A soft set $(A,E)$ is soft $\omega$-open if and only if $(F,E) \subseteq (A,E)^0$ whenever $(F,E)$ is soft semi-closed and $(F,E) \subseteq (A,E)$

**Proof:** Let $(A,E)$ be a soft $\omega$-open set in $X$. Let $(F,E)$ be soft semi-closed set such that $(F,E) \subseteq (A,E)$. Then $(A,E)^c \subseteq (F,E)^c$ where $(F,E)^c$ is soft semi-open. $(A,E)^c$ is soft $\omega$-closed implies $(A,E)^c \subseteq (F,E)^c$ i.e. $(A,E)^0)^c \subseteq (F,E)^c$. That is $(F,E) \subseteq (A,E)^0$.

Conversely Suppose $(F,E)$ is soft semi-closed and $(F,E) \subseteq (A,E)$ Also $(F,E) \subseteq (A,E)^0$. Let $(U,E)^c \subseteq (A,E)$ where $(U,E)^c$ is soft semi-closed. By hypothesis $(U,E)^c \subseteq (A,E)^0$. That is $(A,E)^0)^c \subseteq (U,E)$. i.e. $(A,E)^c \subseteq (U,E)$. This implies that $(A,E)^c$ is soft $\omega$-closed. Hence $(A,E)$ is soft $\omega$-open. $\square$

**Theorem 4.3.** If $(A,E)^0 \subseteq (B,E) \subseteq (A,E)$ and $(A,E)$ is soft $\omega$-open then $B$ is soft $\omega$-open.

**Proof:** $(A,E)^0 \subseteq (B,E) \subseteq (A,E)$ implies $(A,E)^c \subseteq (B,E)^c \subseteq (A,E)^c$ and $(A,E)^c$ is soft $\omega$-closed. By the proposition 3.9 $(B,E)^c$ is soft $\omega$-closed. Hence $(B,E)$ is soft $\omega$-open. $\square$

**Theorem 4.4.** If $(A,E)$ and $(B,E)$ are soft $\omega$-open in $X$ then $(A,E) \cap (B,E)$ is also soft $\omega$-open.

**Proof:** Since $(A,E)$ and $(B,E)$ are soft $\omega$-open their relative complements are soft $\omega$-closed sets and by the Theorem 3.8 $(A,E)^c \cup (B,E)^c$ is soft $\omega$-closed. Hence by The proposition 2.15 $(A,E) \cap (B,E)$ is soft $\omega$-open. $\square$

**Theorem 4.5.** A soft set $(A,E)$ is soft $\omega$-open in $X$ if and only if $(G,E) = \tilde{X}$ whenever $(G,E)$ is soft semi-open and $(A,E)^c \subseteq (G,E)$.

**Proof:** Let $(A,E)$ is soft $\omega$-open and $(G,E)$ is soft semi-open with $(A,E)^0 \cup (A,E)^c \subseteq (G,E)$. Therefore $(G,E)^c \subseteq ((A,E)^0) \cup ((A,E)^c) = (A,E)^c - (A,E)^c$. Since $(A,E)^c$ is soft $\omega$-closed and $(G,E)^c$ is soft semi-closed by the Theorem 3.11 $(G,E)^c = \tilde{\emptyset}$. Therefore $(G,E) = \tilde{X}$.

Conversely suppose that $(F,E)$ is soft semi-closed and $(F,E) \subseteq (A,E)$. Then $(A,E)^0 \cup (A,E)^c \subseteq (A,E)^0 \cup (F,E)^c = \tilde{X})$. It follows that $(F,E) \subseteq (A,E)^0$. Therefore $(A,E)$ is soft $\omega$-open by The theorem 4.2. $\square$

**Theorem 4.6.** Let $(X,\tilde{\tau},E)$ be a soft topological space and $Y \subseteq Z \subseteq X$ are non-empty subsets of $X$. If $Y$ is a soft $\omega$-open set relative to $(Z,\tilde{\tau}_Z)$ and $Z$ is a soft $\omega$-open set relative to $X$, then $Y$ is soft $\omega$-open relative to $X$. 


Proof: Let $(F,E)$ be a soft semi-closed set in $X$ and $(F,E) \subseteq \tilde{Y}$. Then $(F,E) \cap \tilde{Z}$ is soft semi-closed set relative to $(Z,\tilde{\tau}_Z)$. But in $\tilde{Y}$ is soft $\omega$-closed set in $X$ then $(A,E)$ is soft compact.

Theorem 5.1. Let $(A,E)$ be soft $\omega$-closed set relative $(Z,\tilde{\tau}_Z), (F,E) \subseteq (A,E)_{\tilde{Z}}{0}$, where $(A,E)_{\tilde{Z}}{0}$ is the soft open set relative $(Z,\tilde{\tau}_Z),(F,E) \subseteq (G,E) \cap \tilde{Z} \subseteq (A,E)$. Since $\tilde{Z}$ is soft $\omega$-closed relative to $X$, $(F,E) \subseteq (\tilde{Z}){0} \subseteq Z$. Therefore $(F,E) \subseteq (\tilde{Z}){0} \cap (G,E) \subseteq Z \cap (G,E) \subseteq Y$. It follows that $(F,E) \subseteq (Y){0}$. Hence then $\tilde{Y}$ is soft $\omega$-open relative to $X$. 

Theorem 4.7. A soft set $(A,E)$ is soft $\omega$-closed if and only if $\overline{(A,E)} - (A,E)$ is soft $\omega$-open.

Proof: Suppose that $(A,E)$ is soft $\omega$-closed. Let $(F,E) \subseteq \overline{(A,E)} - (A,E)$ where $(F,E)$ is soft semi-closed. By the Theorem 3.11 $(F,E) = \phi$. Therefore $(F,E) \subseteq ((A,E) - (A,E))^0$. By the Theorem 4.2 $(A,E) - (A,E)$ is soft $\omega$-open.

Conversely let $(A,E) \subseteq (G,E)$ where $(G,E)$ is a soft semi-open set. Then $(A,E) \cap (G,E)^c \subseteq (A,E) \cap (A,E)^c = (A,E) - (A,E)$. Since $(A,E) \cap (G,E)^c$ is soft semi closed and $(A,E) - (A,E)$ is soft $\omega$-open, it follows by the Theorem 4.2 $(A,E) \cap (G,E)^c \subseteq ((A,E) \cap (A,E)^c)^0 = ((A,E) - (A,E))^0 = \phi$. Hence $(A,E)$ is soft $\omega$-closed.

Theorem 4.8. For a soft subset of a soft topological space the following are equivalent.

(i) $(A,E)$ is soft $\omega$-closed.

(ii)$\overline{(A,E)} - (A,E)$ contains only null soft semi-closed set.

(iii)$\overline{(A,E)} - (A,E)$ is soft $\omega$-open.

Proof: It follows from the theorems 3.11 and 4.7

5. Applications

Theorem 5.1. Let $(X,\tilde{\tau},E)$ be a soft compact topological space .If$(A,E)$ is a soft $\omega$-closed set in $X$ then $(A,E)$ is soft compact.

Proof: Let $\mathcal{C} = \{(F_i,E) : i \in I\}$ be a soft open cover of $(A,E)$. Since $(A,E)$ is soft $\omega$-closed, $(A,E) \subseteq \bigcup_{i \in I} (F_i,E)$. From the Theorem 2.18 $(A,E)$ is soft compact and hence $(A,E) \subseteq (A,E) \subseteq ((F_1,E) \cup (F_2,E) \cup ...(F_n,E))$ where $(F_i,E) \in \mathcal{C}$ for $i = 1,2,..n$. Hence $(A,E)$ is soft compact.

Theorem 5.2. Let $(X,\tilde{\tau},E)$ be a soft topological space . $Y$ be a nonempty subset of $X$ and if $\tilde{Y}$ be a soft $\omega$-closed set in $X$ then $(Y,\tilde{\tau}_Y)$ is soft normal.

Proof: Let $(A,E)$ and $(B,E)$ be soft closed sets in $X$ and $(\tilde{Y} \cap (A,E)) \cap (\tilde{Y} \cap (B,E)) = \phi$ This implies that $\tilde{Y} \subseteq ((A,E) \cap (B,E))^c \in \tilde{\tau}$ and hence $\tilde{Y} \subseteq ((A,E) \cap (B,E))^c$. Thus $(\tilde{Y} \cap (A,E)) \cap (\tilde{Y} \cap (B,E)) = \phi$. Since $X$ is soft normal there
exists disjoint soft open sets \((G, E)\) and \((U, E)\) such that \((\overline{Y} \cap (A, E)) \subseteq (G, E)\) and \((\overline{Y} \cap (B, E)) \subseteq (U, E)\). Hence it follows that \((\overline{Y} \cap (A, E)) \subseteq \overline{Y} \cap (G, E)\) and \((\overline{Y} \cap (B, E)) \subseteq \overline{Y} \cap (U, E)\). Hence \((Y, \overline{\tau_Y})\) is soft normal. \(\Box\)

6. Conclusion

The class of soft \(\omega\)-closed sets lies between the the class of soft closed sets and the class of soft g-closed sets. The union of two soft \(\omega\)-closed sets is soft \(\omega\)-closed. The necessary and sufficient condition for a soft set to be soft \(\omega\)-closed are derived. The soft \(\omega\)-closed set concept has been extended to subspaces. As an application it has been proved that a soft \(\omega\)-closed set in a soft compact space is also soft compact.

References


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